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LETTER TO THE EDITOR

Effect of traffic accident on jamming transition in traffic-flow model

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Abstract. A cellular atomaton (CA) model is presented to simulate the traffic jam induced by a traffic accident. We investigate the effect of a traffic accident on the dynamical jamming transition in the traffic-flow model. The dynamical jamming transition separates the moving phase in which all cars are moving and the jamming phase in which all cars are stopped. By the use of computer simulation, it is shown that the dynamical jamming transition occurs at lower density of cars with increasing delay time of a car passing over the position of the traffic accident. The phase diagram representing the moving phase and the jamming phase is shown. We also show the anisotropic effect of densities of cars on the dynamical jamming transition.

Recently, traffic problems have attracted considerable attention. The traffic-flow simulations based on various hydrodynamic models have provided much insight [1]. However, the computer simulation of traffic flow in a whole city is a formidable task since it involves many degrees of freedom. The cellular automaton (CA) models have been increasingly used in the simulations of complex physical systems [2–4]. The CA models provide only some general qualitative problems of the complex system while in other cases useful quantitative information can be obtained.

Very recently, Biham, Middleton and Levine [5] have proposed a simple CA model (BML model) to describe a traffic flow in two dimensions. The traffic-flow model is given by a three-state CA on the square lattice. Each site contains either a car moving upwards, a car moving to the right, or empty. They have found that a dynamical jamming transition occurs at the critical density $p = p_c$ ($p_c \approx 0.3$–0.4) with increasing density of cars. The dynamical jamming transition separates the low-density moving phase in which all cars move and the high-density jamming phase in which all cars are stopped. In real traffic-flow systems, the traffic jam is frequently induced by a traffic accident. The traffic accident prevents cars from passing over the point of the accident. It will be easy for the traffic jam to occur due to the traffic accident. There is an open question as to whether or not the car accident changes the jamming transition. A simple CA model taking into account the traffic accident has been unknown until now.

In this letter, we extend the BML model to take into account a traffic accident. We investigate the effect of the traffic accident on the dynamical jamming transition. We show the phase diagram representing the moving phase and the jamming phase. Also, we show the anisotropic effect of densities of cars on the jamming transition. We find that the traffic accident has an important effect on the jamming transition.

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We describe an extended version of the BML model showing a traffic jam in two dimensions. The model is defined on the square lattice of $n \times n$ sites with periodic boundary conditions. The traffic-flow model is a three-state CA. Figure 1 shows the schematic illustration of the CA model. Each site contains either an arrow pointing upwards, an arrow pointing to the right, or empty. The arrow pointing upwards represents the car moving up. The arrow pointing to the right represents the car moving to the right. The car accident can occur on any site. However, the traffic flow does not change because of the position of the car accident because of the periodic boundary condition. We put the position of the car accident at the centre of the square lattice. It is hard for a car to pass over the position of the car accident. We model the traffic-flow condition of the traffic accident as follows. When a car moving up reaches the site at the centre of the square lattice, it can pass over after T time steps. The time constant T means the delay time of a car flowing over the position of the car accident.

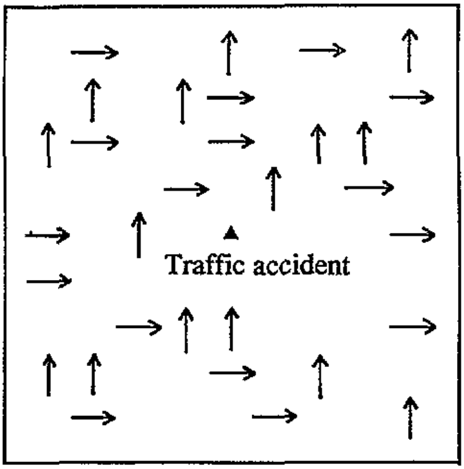

Figure 1. Schematic illustration of the cellular automaton model for the jamming transition induced by the traffic accident in traffic flow. The arrows pointing up represent the cars moving up. The arrows pointing to the right represent the cars moving to the right. The position in which the traffic accident happened is indicated by the solid triangle.

The limiting case of $T = 1$ corresponds to the original BML model without the traffic accident. In the limiting case of $T \to \infty$, a crashed (or stopped) car is introduced on the site at the centre of the square lattice. Moving cars are stopped as soon as they reach the cluster of jammed cars. One after the other, moving cars are stopped by the traffic jam. The traffic jam propagates from the crashed car to cover all available space. The spreading of the traffic jam is similar to the crystal growth initiated by a seed. In the crystal growth process, a single seed triggers the crystal growth. Generally, crystal growth may be difficult without a single seed. The crystallization process strongly depends on the seed. The crystallization processes are different in the cases with or without a seed. Similarly, it will be difficult for a traffic jam to occur, without a car accident, for a low density of cars. As soon as a crashed (or stopped) car is introduced, a traffic jam propagates into all available space. Therefore, there is an analogy between the traffic jam and the crystal growth. The limiting case of $T \to \infty$ is equivalent to the ballistic deposition model at a finite concentration for thin films [6].

The traffic flow, except the position of the car accident, is described in terms of the same CA model as the BML model. The dynamics is controlled by a traffic light, such that the arrows pointing to the right move only during odd time steps and the arrows pointing up move during even time steps. During odd time steps, each right arrow moves one step to the right unless the site on its right-hand side is occupied by another arrow (which can be either an up or right arrow). If an arrow is blocked by another arrow it does not move, even if during the same time step the blocking arrow moves out of the site.

In this model, the traffic problem is reduced to its simplest form while the essential features are maintained. These features include the simultaneous flow in two perpendicular directions of cars which cannot overlap. Furthermore, this model possesses such a property that it is hard for a car to pass over the position of the traffic accident. We perform the computer simulation of the traffic-flow model and study the effect of the traffic accident on the traffic jam. The effect of the car accident on the traffic flow is included into the delay time T. Initially, cars are randomly distributed at the sites on the square lattice. Due to the periodic boundary conditions, the total number of arrows of each type is conserved. Moreover, the total number of up arrows in each column and the total number of right arrows in each row are conserved, giving rise to $2n$ conservation rules. The density of right (up) arrows is given by p_x (p_y) where the density p of cars is $p=p_x+p_y$.

We have performed computer simulations of the CA model starting with an ensemble of random initial conditions for the system size $n=10$–50 and the density $p=0.0$–1.0 of cars. Each run is obtained after 10 000 time steps. We have obtained the mean velocities of cars by averaging over 100 runs. Figure 2 shows the plot of the mean velocity $\langle v\rangle(=\langle v_x\rangle=\langle v_y\rangle)$ of cars against the density p of cars for the delay time $T=1$–20 with $p_x=p_y=p/2$ and $n=50$. In the case of $p_x=p_y$, the mean velocity $\langle v_x\rangle$ of cars moving to the right equals the mean velocity $\langle v_y\rangle$ of cars moving up. The mean velocity $\langle v_x\rangle$ of right arrows in a unit time interval is defined to be the number of successfully moving right arrows divided by the number of right arrows. The mean velocity $\langle v_y\rangle$ of up arrows in a unit time interval is defined to be the number of successfully moving up arrows divided by the number of up arrows. The velocity $\langle v\rangle$ has maximum value $\langle v\rangle=1$, indicating that the arrow is never blocked, while $\langle v\rangle=0$ means that the arrow is stopped and never moves at all. The velocity of $T=1$ indicated by white circles corresponds to that in the original BML model. The mean velocity $\langle v\rangle$ begins to decrease

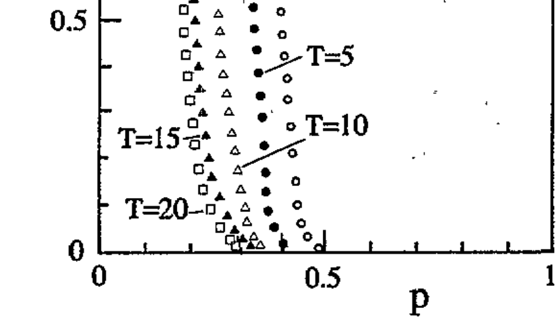

Figure 2. The plot of the mean velocity $\langle v\rangle$ of cars against the density p of cars for the delay time $T=1$, 5, 10, 15 and 20.

sharply near $p \approx 0.30$ and becomes zero at $p \approx 0.50$. The dynamical jamming transition occurs at $p \approx 0.50$. The jamming transition separates between the low-density moving phase in which all cars move and the high-density jamming phase in which all cars are stopped. In the cases of the traffic accident ($T > 1$), the velocity $\langle v \rangle$ begins to decrease at lower density with increasing delay time T. The jamming transition also occurs at lower density of cars with increasing T. The jamming transition depends on the system size n. In figure 3, we plot the transition point p_c against the inverse $1/n$ of the system size for various values of the delay time T. Here we define the transition point p_c as the value of p in which $\langle v \rangle = 0.5$. By extrapolation, we estimate the value

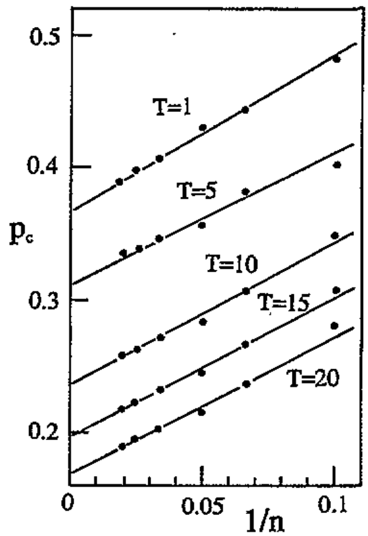

Figure 3. The plot of the jamming transition point p_c against the inverse $1/n$ of the system size for extrapolation.

of the transition point in the limit of an infinite system size. Figure 4 shows the phase diagram between the density p and the inverse $1/T$ of the delay time. The phase diagram is obtained from the estimated value of the transition point in the infinite limit. Also, the black circles indicate the transition point for $n = 50$. The region on the left-hand side of the solid curve indicates the moving phase in which all cars move. The region on the right-hand side of the solid curve indicates the jamming phase in which all cars are stopped. With increasing delay time T, it is easy for a traffic jam to occur. In the limit of $T \to \infty$, the traffic jam occurs at $p \to 0$. Moving cars are stopped as soon as they reach the position of the traffic accident. One after the other, moving cars are stopped by the jammed cars. The jammed cars grows with increasing time. The traffic accident induces the traffic jam even at a low density of cars. The car accident has an important effect on the jamming transition.

We study the anisotropic effect of densities of cars on the dynamical jamming transition. In the anisotropic case in which $p_x \neq p_y$, the density of cars moving up is defined by $p_y = pf$ and the density of cars moving to the right is defined by $p_x = p(1-f)$ where p is the total density of cars and f is the fraction of cars moving up. Figure 5 shows the plots of the mean velocities $\langle v_x \rangle$ and $\langle v_y \rangle$ of cars against the density p of cars for the fraction $f = 0.8$ and 0.9 with delay time $T = 10$ and the system size $n = 50$. The data points are compared with those ($T = 10$) indicated by white triangles in figure

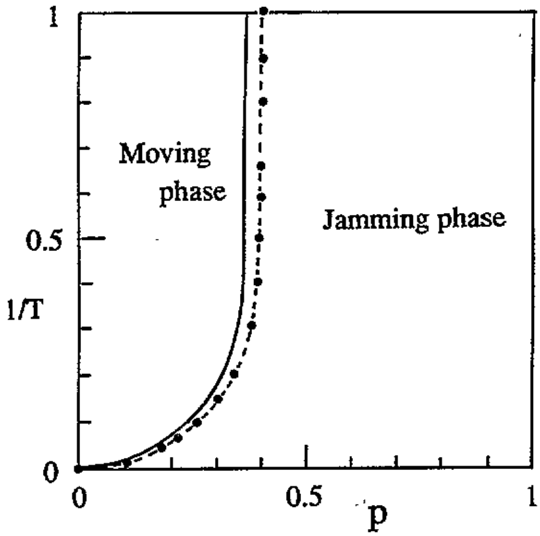

Figure 4. The phase diagram of the dynamical jamming transition. The density p of cars at the transition point is plotted against the inverse $1/T$ of the delay time. The solid curve indicates the jamming transition points in the limit of the infinite system size. The solid circles indicate the transition point for the system size $n=50$.

2 which represent the mean velocity $\langle v\rangle=\langle v_x\rangle=\langle v_y\rangle$ in the isotropic case ($f=0.5$). In the anisotropic cases, the velocity $\langle v_x\rangle$ of cars moving to the right becomes different from the velocity $\langle v_y\rangle$ of cars moving up. By increasing the fraction f ($f>0.5$), the difference between $\langle v_x\rangle$ and $\langle v_y\rangle$ becomes larger and larger. The tail of the velocity distribution becomes longer and longer with increasing f. The jamming phase appears with higher density than the isotropic case of $f=0.5$. The jamming transition occurs at higher density than that of the isotropic case with increasing f. In the limit of $f=1.0$ ($f=0.0$), the jamming phase with $\langle v\rangle=0.0$ disappears and the dynamical jamming transition does not occur. Figure 6 shows the phase diagram of the dynamical jamming transition for the system size $n=50$. The region on the left-hand side of the transition point represents the moving phase in which all cars move. The region on the right-hand side of the transition point represents the jamming phase in which all cars are stopped.

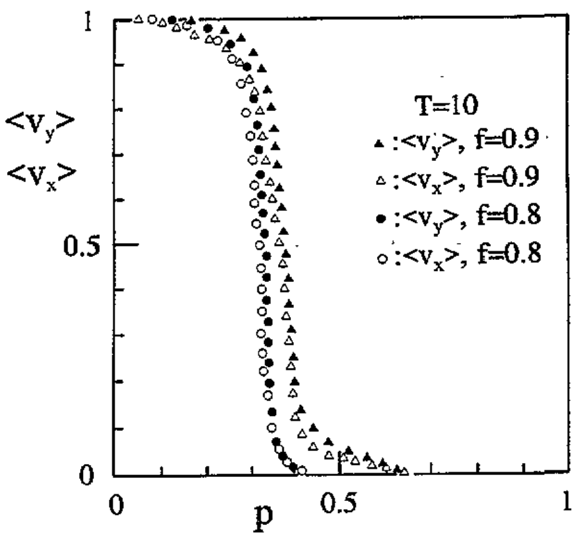

Figure 5. The plot of the mean velocity $\langle v_x\rangle$ of cars moving to the right and the mean velocity $\langle v_y\rangle$ of cars moving against the density p of cars for the anisotropic cases of $f=0.8$ and 0.9 with $T=10$.

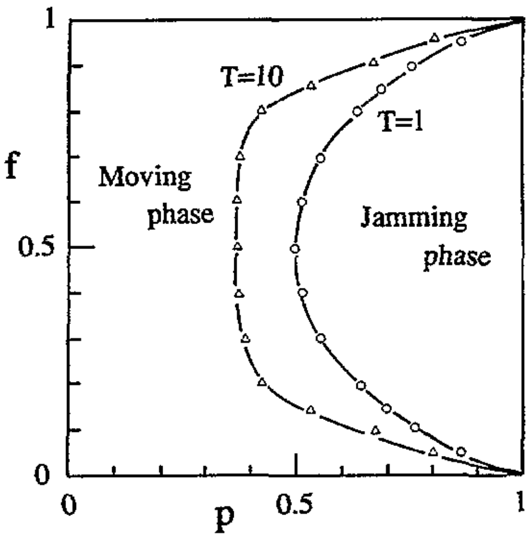

Figure 6. The phase diagram of the jamming transition representing the anisotropic effect. The density p of cars at the transition point is plotted against the fraction f for $T=1$ and 10.

The density p of cars at the transition point is plotted against the fraction f for the cases of $T=1$ and 10. The data of $T=1$ means the traffic jams without the car accident. The jamming transition shifts to the region of low density by the effect of the traffic accident. The anisotropic densities of cars also have an important effect on the jamming transition. It is hard for right arrows to block up arrows in the anisotropic case $f>0.5$. Therefore, the jamming transition occurs at higher density with an increase in the anisotropic effect.

In summary, we extend the BML model to take into account the traffic accident. We investigate the effect of the traffic accident on the traffic jam by using computer simulation. We find the phase diagram representing the jamming transition. We also show the anisotropic effect of densities of cars on the jamming transition.

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